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The Mechanical Equivalent of Heat

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Phil. Trans. R. Soc. Lond. A 1928 **227**, 63-91
doi: 10.1098/rsta.1928.0003

III. *The Mechanical Equivalent of Heat.*

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(*Communicated by Sir* Ernest Rutherford, *O.M., P.R.S.*)

(Received January 31,—Read February 17, 1927.)

Introduction.

Dr. J. K. Roberts and the senior author of this paper began experiments upon the mechanical equivalent of heat* in 1918. Mr. Roberts left Australia in 1920, and in that year he published an account of the principles of the design of the induction dynamometer which we have used. The experiments have been continued by the authors of this paper, but so many difficulties have had to be overcome to attain high accuracy as to make the investigation a prolonged one.

What may be called the electrical equivalent of heat has been the subject of the well-known investigations of Griffiths, Schuster and Gannon, Callendar and Barnes, W. R. and W. E. Bousfield, and Jaeger and Steinwehr. One† of us has given a critical discussion of these experiments, and has corrected the previously published results to the thermodynamic scale of temperature, and the electrical units used to their now accepted absolute values. The direct determination of J has received much less attention; for, in addition to Joule's original experiments, there are only those of Rowland, and Reynolds and Moorby.

As the mechanical equivalent of heat is one of the fundamental constants of physics, it is desirable that it should be known with the highest precision attainable with the present developments of physical technique. It can then be compared with the electrical equivalent of heat.

Summary.

As mentioned above, there are only two accurate direct determinations for the mechanical equivalent of heat. Both are open to criticism in the light of modern standards of accuracy, Rowland's particularly, on account of his thermometry, and Reynolds and Moorby's because the heat losses were uncertain, and the steadiness of conditions essential to continuous flow calorimetry was not realised.

In the experiments which we have made continuous flow calorimetry has been used, although it is not claimed that as a calorimetric method it has any very marked superiority

* 'Proc. Roy. Soc. Vict.,' vol. 32, p. 156 (1920).
The symbol "J" will be used for "mechanical equivalent of heat."
† Laby, 'Proc. Phys. Soc.,' vol. 38, p. 169 (1926).

over the rising temperature method. An induction dynamometer (see figs. 1 and 2) similar to the squirrel-cage induction motor has been used. In this dynamometer a copper calorimeter is situated in a rotating magnetic field, which causes a couple to act on and heats the calorimeter or stator. A rotating electromagnet produces the rotating field.

The input of energy in an experiment is proportional to the mean couple acting on the calorimeter (or stator) and to the number of turns of the rotor. These are measured in the usual way, as, for example, by ROWLAND, except that the measurement of the couple has been made much more precise.

The heat generated is continuously absorbed by water, which flows through the copper tubes forming part of the stator. The mass of this water in an experiment and its rise of temperature give the heat developed, uncorrected for the friction of the water and heat losses. It is shown that the value of J is given by the simple expression :

$$J = \pi nmg\, d_\theta f \,.\, [W\{(\theta_2 - \theta_1) - v\} + L]^{-1},$$

where $\pi nmg\, d_\theta f$ is the measure of the work done during an experiment, and $W\{(\theta_2-\theta_1)-v\}$ is the measure of the heat abstracted by the water and L of the heat lost, and v represents the increase of temperature due to the viscosity of the water.

The heat losses proved very difficult to determine in the earlier experiments, which extended over a period of several years. The induction dynamometer was designed to dissipate about 250 watts in a comparatively small calorimeter so that the heat lost might be a small fraction of that developed.

To make the thermal insulation as perfect as possible, the calorimeter was enclosed in a Dewar flask, and the thermometers were vacuum jacketed. In spite of this insulation heat was lost from the upper end of the calorimeter through the mouth of the flask to the air. Prolonged attempts to measure this loss having failed, it was made vanishingly small by adjusting the air temperature to be closely that of the upper end of the calorimeter. The loss through the vacuum jackets was less than 0·001 per cent., and the other heat losses 0·003 per cent. of the heat generated in an experiment. Previous observers of J have had heat losses of from one to six per cent.

In a determination of J a high degree of steadiness of conditions is essential, but difficult to realise. It is specially important in our experiments that the intensity and rate of rotation of the magnetic field, which determine the couple, should both be constant, if possible, to 1 in 10,000. We realised this degree of steadiness in respect to the intensity of the magnetic field, but not always in the speed. The steadiness in the rise of temperature depends on these quantities and upon the steadiness of the rate of flow.

The side wheels over which fine tungsten wires pass from the weights, which give the couple, to the torsion wheel were made with agate knife-edges in order to eliminate friction. The knife-edges were accurately centred and the wheels carefully balanced.

The thermometers used are of platinum differentially connected so that a single bridge reading gives the temperature rise. The scale of temperature used is the platinum scale, as fixed by using for the boiling point of sulphur 444·55° C., its value on the thermodynamic scale. The thermometers have been remarkably stable; one has not changed its constants in five years. The main thermometric difficulties have been in the accurate reproduction of the ice, steam, and sulphur points and in barometry. At the room temperature the accuracy of the platinum thermometers is believed to be better than 1/1000° C.

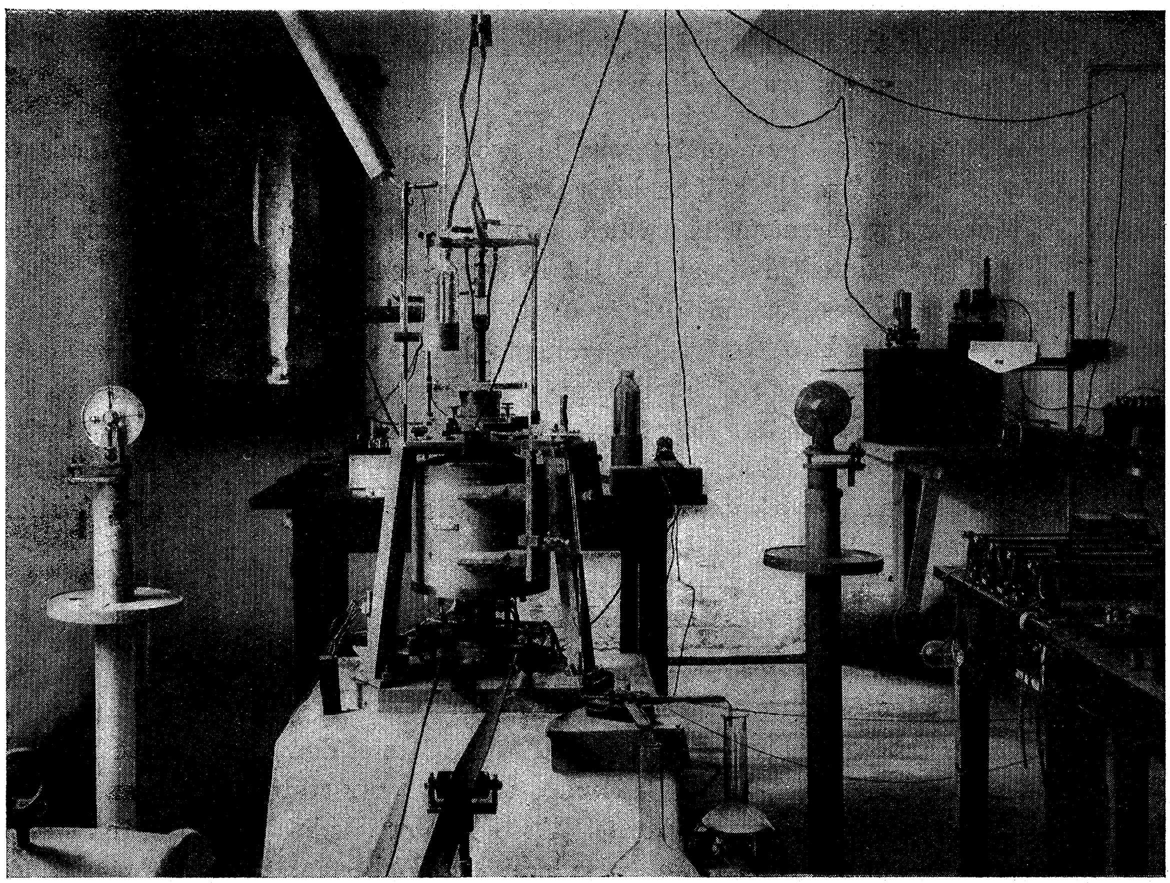

FIG. 1.—*Mechanical Equivalent of Heat Apparatus.* Induction dynamometer in centre of photograph.

Twenty-three determinations of J are given at mean temperatures from 15·88° C. to 20·50° C. The weighted mean of these experiments is $4 \cdot 1841 \, . \, 10^7$ erg for the 16·7° calorie with a probable error of 0·0001. Using CALLENDAR's values for the temperature variation of the specific heat of water, this gives 15° C. calorie $= 4 \cdot 1860 \, . \, 10^7$ erg, and 20° C. calorie $= 4 \cdot 1809 \, . \, 10^7$ erg. A critical consideration of the electrical determinations of J shows that this value lies within the range of the best of those determinations.

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Previous Direct Experiments.

The accuracy of the direct experiments of ROWLAND and REYNOLDS and MOORBY has been discussed by GRIFFITHS,* AMES,† CALLENDAR,‡ and LUTHER and SCHEEL.§

In the method of calorimetry used by ROWLAND (which may be called the method of mixtures) the main difficulties are :—

(1) To determine the *rate* of rise of temperature with the requisite accuracy, for the lag of the thermometer behind the actual temperature is dependent on the rate of rise, and is different initially and finally ;
(ROWLAND does not consider this source of error.)

(2) To determine the water equivalent of the calorimeter ;

(3) To attain the necessary steadiness.

Although ROWLAND'S thermometry was in accordance with the best methods of 1879, AMES says his "method of making thermometric readings is liable to serious error." The mercury thermometers which he used were studied eighteen years later by DAY,|| who recalculated his results to the hydrogen scale of temperature.

GRIFFITHS states that errors of 0·03° to 0·04° C. occurred in ROWLAND'S individual observations. In the light of our own experiments there is another criticism to be made. It is of the two pulleys over which silk tapes passed from the hanging weights to the torsion wheel of the calorimeter. These have to be most carefully designed¶ and tested if they are to be free from friction. No description of these pulleys is given by ROWLAND. Dr. ROBERTS' experiments with journal and pivot bearings showed them to have friction too large and variable to allow of their use.

To avoid the difficulties of exact temperature measurement, REYNOLDS and MOORBY used the continuous flow method of calorimetry over the range (approximately) 0° to 100° C. In such calorimetry it is assumed that different parts of the calorimeter are stationary in temperature. To realise this, very great steadiness in the rate of rotation of the paddles, the applied couple, and in the rate of flow of the water through the calorimeter, etc., are essential. REYNOLDS and MOORBY state that air and steam bubbles were liberated in their experiments. As the water was heated nearly to boiling, and a Froude brake was used by them, these bubbles would be (as we have found under much more favourable conditions) a serious cause of unsteadiness. Further, the bubbles gave rise to another uncertainty, as to the amount of energy used in forming the steam in them.

* 'Phil. Trans.,' A, vol. 184, p. 361 (1893); "Mechanical Equivalent of Heat," 'Dict. App. Physics,' vol. 1, p. 477.

† 'Rapp. Cong. Internat. de Physique,' vol. 1, p. 178 (1900).

‡ 'Phil. Trans.,' A, vol. 199, p. 55 (1902).

§ 'V. Deut. Phys. Ges.,' vol. 10, p. 584 (1908).

|| DAY, 'Phil. Mag.,' vol. 46, p. 1 (1898), and 'Phys. Rev.,' p. 193, vol. 6 (1898).

¶ See p. 74 of this paper.

The outflow temperature in REYNOLDS and MOORBY'S experiments varied as much as 4·9° F. in 120 seconds, and ordinarily it changed so rapidly that it could not be read to better than 0·1°. Their speed varied by as much as 3·3 per cent. in 30 mins., with normal variations of 0·3 per cent. The Froude brake used by REYNOLDS and MOORBY had a maximum capacity of 100 H.P. The static balance could not be determined to better than to 40 ft. lbwgt. which is 10 per cent. of the couple in certain runs.* The accuracy of their result depends upon the error arising from this uncertainty, eliminating when heavy and light runs are combined. In some experiments about 1/400 of the heat generated was lost through 4-inch steel shaft of the brake dynamometer. The means used to eliminate this loss were very uncertain. No doubt such difficulties would be very hard to exclude from experiments carried out on the scale of those of REYNOLDS and MOORBY.

Their result for J in erg per mean cal. can only be expressed in erg per 20° cal., if the relation of these two calories is known. The disagreement in the observed values of the specific heat of water over the range 60° to 100° C. makes the relation somewhat uncertain.

Briefly, then, there are only two direct determinations of J, and in one of these the temperature scale is uncertain, and in the other the unit of heat. The electrical equivalent of heat has been more frequently investigated, and its value is more certain than that of J.

Principles of the Experiment.

It will be convenient to explain further the principle of the experiment before describing in detail the experiments we have made. The continuous flow method of calorimetry appeared to possess on the whole advantages over the method of mixtures. When applied to the direct determination of J, two serious difficulties have to be overcome, namely,

(1) to attain the required steadiness in all the conditions of the experiment, and

(2) to find the heat losses with sufficient accuracy.

The dynamometer used may be called an induction dynamometer, on account of its similarity to an alternating current induction motor.† There is, however, only a general similarity, and to prevent confusion the following differences are stated. The squirrel-cage rotor of the motor becomes in the dynamometer the stator and calorimeter, and the stator of the motor with its field windings excited by an alternating current is replaced in the dynamometer by an electromagnet (excited by direct current) which rotates about a vertical axis and will be called the rotor.

The essentials of the dynamometer are shown in fig. 2. The electromagnet NS rotates about a vertical axis OZ, the magnet being excited by means of a battery to which it is connected through slip rings R. The stator, very similar in construction to the rotor of

* In our experiments the uncertainty is 1/30,000 of the maximum couple.

† A rotating magnetic field produced by alternating currents would not be satisfactory on account of the difficulty of making its axis coincide with that of the stator.

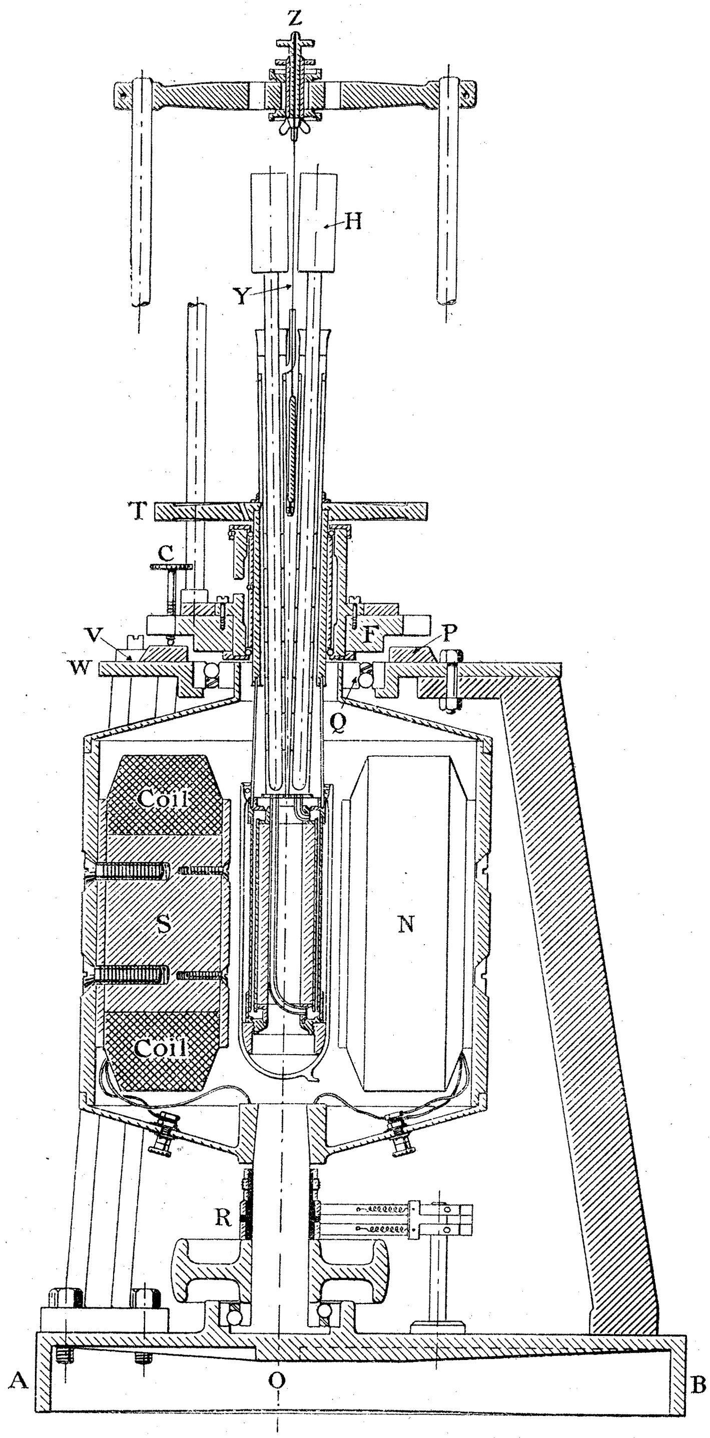

FIG. 2.—*Induction Dynamometer and Calorimeter in Section.* AB, base plate. OZ, vertical axis of rotation. R, slip rings connected to coils of electromagnet NS. Vacuum flask enclosing calorimeter. The plate F carries the ball bearing of the stator, which is levelled and centred by the slides V and screws C. T, torsion wheel. Y, torsion wire. H, thermometer, which is enclosed in a vacuum jacket as shown in fig. 3.

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an induction motor, is mounted on ball bearings and torsion wire so as to be free to rotate about a vertical axis, except in so far as it is constrained by the couple applied to the torsion wheel attached to it. The rotating magnetic field induces currents in the copper tubes of the stator, resulting in a couple acting upon it and heat being generated in it when it is held stationary. The copper tubes are water-cooled.

For this type of dynamometer to be suitable for the determination of the mechanical equivalent of heat it must satisfy certain conditions, which are—(1) The rotating magnetic field must be of constant intensity; (2) the axis about which the magnetic field rotates must be parallel to the axis of the stator, but have no component of motion in the direction of that axis. If these conditions are fulfilled, a couple will act upon the stator corresponding fully to the heat produced. If, for example, the magnetic lines of force had a component of motion parallel to the axis of the stator, heat would be generated, but there would be no corresponding couple about that axis.

Induction dynamometers have been used according to FREUND* for experiments on the mechanical equivalent of heat, but nothing is described by these workers which makes it efficient or accurate for that purpose.

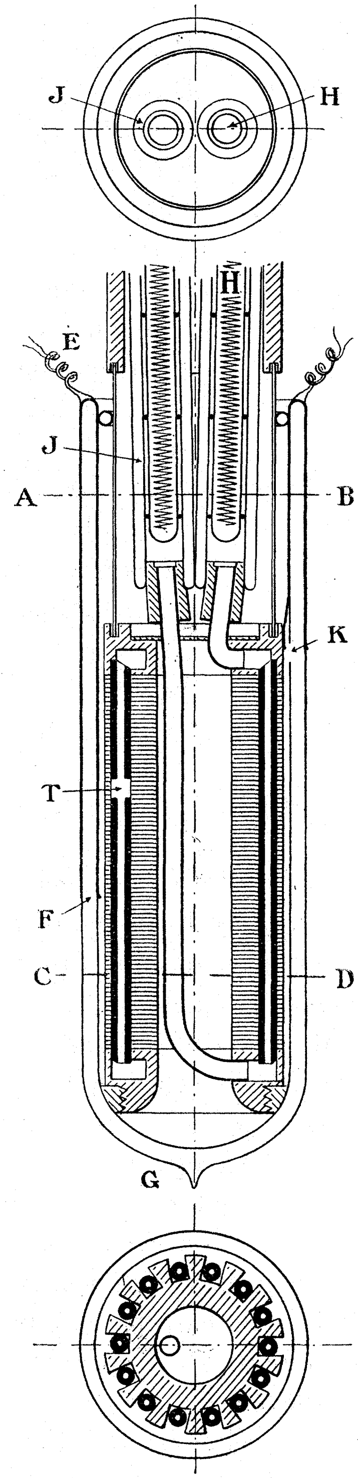

FIG. 3.—*Stator and Calorimeter.* G, vacuum flask. The inlet water flows past the thermometer H to the upper annualar channel, down fourteen copper tubes T to the lower annular channel, thence to the outlet thermometer at J. J, vacuum jacket. F, thermo-couple junctions with leads at E. K, thermo-couple junctions, one junction being in contact with top of calorimeter. Section through AB shown above, through CD below.

Expression for J.

Let

n = number of revolutions of rotor during a run.

m. gm. = total suspended mass, including corrections for overhanging wire, lack of balance of knife-edge wheels, buoyancy of weights, torsion of suspension wire.

* R. FREUND, 'Phys. Zeit.,' vol. 15, p. 817 (1914). FREUND obtained, for the value of the mechanical equivalent, $4\cdot26 \times 10^7$ erg/cal. D'ARSONVAL in 1891 found $4\cdot14$ to $4\cdot19 \times 10^7$.

f = Correcting factor for centering of knife-edge wheels.
d_θ cm. = Diameter of torsion wheel at $\theta°$ C. + diameter of wire.
g = 979·979 the measure of acceleration of gravity at Melbourne University.*
W gm. = Mass of water, corrected for buoyancy.
$\theta_2 - \theta_1$ = Corrected temperature rise.
$v°$ C. = Part of temperature rise due to fluid friction. $v = 0·0016$ in the experiments described.
J = ergs per calorie for a temperature rise θ_1 to θ_2.
L. cal = Heat loss through vacuum flask walls during experiment; this is proportional to the mean reading of a thermocouple during a run. The loss through the thermometer jackets is less than $\frac{1}{10000}$ of W $(\theta_2 - \theta_1)$.

Then

$$\pi nmgd_\theta f = \mathrm{J}[\mathrm{W}\{\theta_2 - \theta_1 - v\} + \mathrm{L}].$$

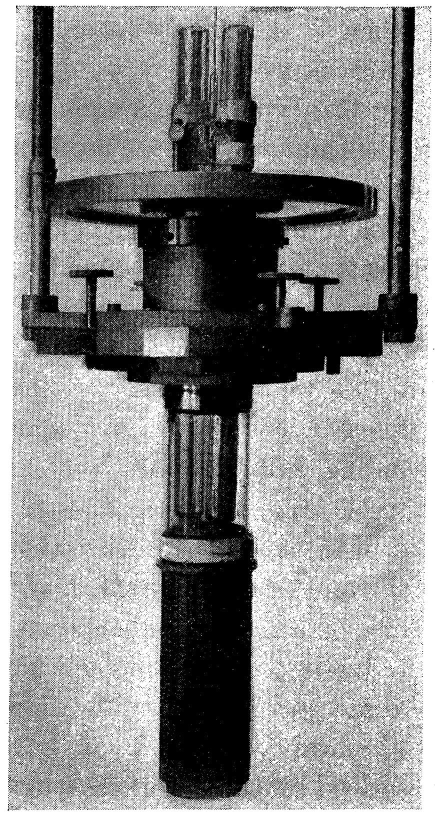

FIG. 4.—*The Stator.* From lower end upwards: squirrel-cage armature, and copper tubes, vacuum jackets, levelling screws, outer case of ball bearing, torsion wheel.

Detailed Description of Apparatus.

The theory of the design of the induction dynamometer has been discussed by Dr. ROBERTS,† and the dynamometer which we have used was designed in the light of his experiments, but the stator differs from the one described by him.

The rotor is shown in fig. 1; it is very similar to the field magnet of a direct-current dynamo, but the magnetising windings are taken to slip rings. The rotor is carried on S.K.F. ball-bearings, mounted on a cast-iron framework (as shown in fig. 2), consisting of a triangular base plate, and three side members which support the upper bearings. The base, which is bolted to a heavy concrete block, is levelled until the axis of the rotor is accurately vertical. To prevent the stator flask being broken, the rotor must be very accurately balanced. The rotor is belt driven by a direct-current motor.

The *stator* is shown in figs. 2 and 4, its design having been determined after two other

* Five different observers have determined the acceleration of gravity at Melbourne relative to that at Potsdam. See KAYE and LABY, 'Physical Constants,' London.

† *Loc. cit.*

designs had been tested.* It will be seen to be very similar to the stator of an induction motor, with the difference that copper tubes are used in place of the usual rods. The cooling water comes in through a tube to one of the brass end rings with an annular channel communicating with each of the fourteen copper tubes, and flows out through the other end ring. The armature is made of stalloy stampings, and consequently most of the heat is generated in the copper tubes. The water cooling is efficient, as the surface of the tubes in contact with it is large.

To connect the stator rigidly to its bearing, and at the same time to insulate it thermally, a glass tube is inserted between the two (see fig. 4). The glass to metal joints, made with sealing wax, held perfectly for some years, but once, on a hot Australian day, when an exceptionally heavy "run" was in progress, the sealing wax yielded, the flask burst, and the stator was thrown into the rotor, setting up very severe centrifugal strains, which wrecked most of the glass parts of the apparatus, including one of the thermometers.

The *ball-bearing*, which carries the stator, is made of large diameter, so that the thermometers can be placed inside it. It consists of two hollow cylinders of case-hardened steel (see fig. 2). Two rings of $\frac{1}{4}$-inch steel balls separate these two cylinders, the upper ring of balls being supported by the upper edge of the sleeve, and the lower ring of balls by a cap. Very great care was taken in the construction of this bearing. After case-hardening, the cylinders were heat-treated to remove strains, ground, and lapped to size, the lapping being controlled by measurements with Johannsen gauges. The bearing, used without lubricant, proved entirely satisfactory, and there was no evidence of friction, provided the working surface were polished and free from dust.

The inner sleeve of the bearing is attached to the torsion wheel, and the wheel and it are supported by a wire, as shown in fig. 2. The top of the wire has several adjustments.

The outer sleeve of the bearing is so supported by a flange F, attached to a plate P, as to enable the stator axis to be made coincident with that of the rotor. The flange is attached to the plate by levelling screws C, and holding-down screws. This enables it to be tilted about either of two horizontal axes. Two pairs of parallel V slides, V, enable the plate P, which bears on another plate W, to be moved in two horizontal directions at right angles and then rigidly clamped in its correct position by four screws.

To *centre the stator* relative to the rotor, the stator unit is lifted out of the rotor, the flask is removed (see fig. 4), and two circular brass rings are put round the stator (its surface is "cylindrical" but for a very slight taper) and pressed home. The stator unit is put back, and it is tilted and moved horizontally until the pole pieces just clear the rings. The stator is then centered. It can be withdrawn, the rings removed, the flask put back, the stator unit replaced, and the centering so found will be recovered.

The *torsion wheel*, T, is of aluminium alloyed with 4 per cent. copper, and is 20·001 cm. diameter at 20° C. A tungsten wire (0·0127 cm. diameter) supporting one of the weights passes over one of the knife-edge wheels to the torsion wheel to which it is attached, the

* Laby and Roberts, 'Roy. Soc. Proc. Vict.,' vol. 32, p. 148 (1920).

other wire being attached to the other side of the torsion wheels. The parts of these wires between the torsion and knife-edge wheels are adjusted to be parallel and horizontal.

Calorimeter.—The flask G, which encloses the stator, is screwed on to the bottom end of it by means of a threaded iron ring which is sealing-waxed into the bottom of the flask. The top of the flask is closed by means of a ring of round rubber, which is pressed into the narrow space between the mouth of the flask and the glass tube which supports the stator. In our preliminary experiments the use of a vacuum flask for this purpose proved difficult. The clearance between the flask, which was not perfectly cylindrical, and the rotor was often not quite 1 mm. Any marked vibration caused the flask to hit the rotor. A number of flasks were lost in this way, until improvements in the bearing of the stator, better balancing of the rotor, and the use of more accurately shaped flasks, reduced the amplitude of vibration to less than the clearance. We are indebted to Messrs. Landers, Frary and Clark and to the Thermos Company for making the special flasks used in this investigation. The vacuum jackets mentioned later which enclosed the platinum thermometers were made by the National Glass Industry. This firm successfully overcame the difficulties of making this class of vacuum vessel.

In the earlier forms of the apparatus the thermometers were placed at some distance from the calorimeter. This gave rise to heat losses between the thermometers and the calorimeter, which we were unable to measure. This defect in the design was overcome by a complete re-design of the induction dynamometer in such a way that the thermometers, insulated with jackets of powdered cork (see fig. 2), were brought within an inch of the calorimeter. Although this change, by considerably reducing the heat losses, improved the calorimeter, it did not remove all difficulties. A number of experiments were made using different power inputs—namely, 20, 40, 70, and 100 watts. When these were analysed it became evident that there was a loss (or gain) of heat from the thermometers and from the top of the calorimeter. The loss from the calorimeter was investigated by attaching to it at the top, middle, and bottom, thermo-junctions. Further experiments on the mechanical equivalent of heat showed that there was a flow of heat from the top of the calorimeter, through the mouth of the vacuum flask surrounding it, to the air.

The heat losses from the thermometers were made vanishingly small by enclosing them in silvered vacuum jackets (see fig. 3). The loss from the top of the calorimeter was made unimportant by (1) increasing the length of the flask enclosing it until the mouth of the flask was well above the top of the calorimeter ; (2) making the temperature of the top of the calorimeter that of the air of the room. The latter was done by bringing the inlet water through the top of the calorimeter. As the metal top was about 2° C. hotter than the water, the air of the room was warmed by the correct amount above the temperature of the inlet water.

A thermo-junction with one couple attached to the top of the calorimeter and the other to the outside wall of the flask (see fig. 3, K) read zero when the temperature of the air had been correctly adjusted. The rotation of the motor kept the air thoroughly stirred.

Water Circulation.—The water-circulation system is shown purely diagrammatically in fig. 5.

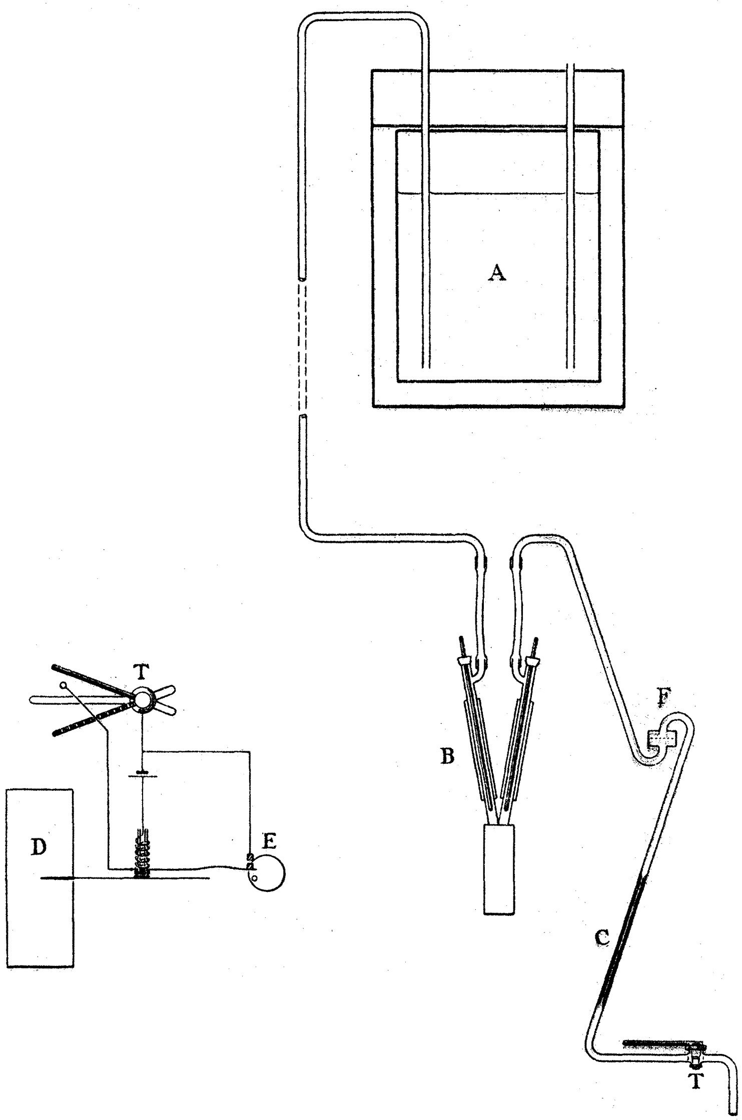

FIG. 5.—*Water Circulation and Chronograph.* A, tank. B, thermometers and calorimeter. C, capillary resistance tube. T, tap and timing contact. D, chronograph drum. E makes contact every 100 turns of the rotor. F, filter.

Distilled water at a constant head about 10 feet and constant temperature is supplied from a 35-litre tank. The temperature of the inlet water is observed by a thermo-junction attached to the inlet platinum thermometer. Rubber tubing is used to give a flexible connection to the calorimeter. Metal rings, with small gaps, were threaded round the thermometers to produce eddies in the water, and prevent it flowing in stream

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lines past the thermometers, and so ensure that the thermometers were at the mean temperature of the water surrounding them. With vacuum jackets and a layer of water about 1 mm. thick round the thermometers there seems to be little possibility of error through lack of uniformity of temperature in the water.

The rate of flow of the water was controlled by the glass capillary C, shown in fig. 5, different capillaries being used to give different rates of flow. The capillary was protected from obstruction by a fine metal gauze filter F.

A two-way tap T, operated by strings, enabled the discharged water to be deflected from the waste vessel to the collecting vessel. It was turned in a fraction of a second, and as it turned it closed the chronograph circuit.

The *motor*, which drives the rotor, is a 1 h.p. 100-volt shunt wound direct-current motor. It was connected to a new Tudor storage battery of ample capacity. The drive was by belt with a glued joint carefully made.

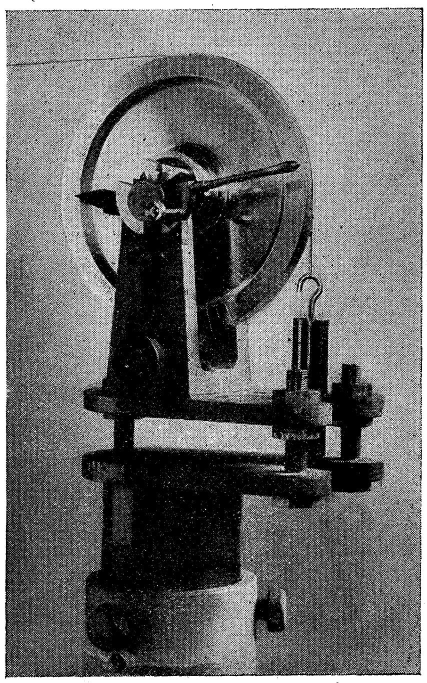

FIG. 6.—*Agate knife-edge Side Wheels.* The other face of a wheel is shown on the left of fig. 1.

Knife-Edge Wheels.—Preliminary experiments with cone, ball and other kinds of bearings showed that, if the couple applied to the stator was to be known to 1 in 10,000, the pulleys (over which wires pass from the hanging weights to the edge of the torsion wheel) had to be carefully designed and tested. Wheels supported by steel knife-edges proving not entirely satisfactory, agate edges were substituted. The wheels are shown in figs. 1 and 6.

The horizontal pull, P, on the wire is not exactly equal to the suspended weight, W, but

$$P/W = 1 + x(\cos\alpha + \sin\alpha) + y(\cos\alpha - \sin\alpha) - (mgh\sin\alpha)/r,$$

where x, y specify the small distance of the knife-edge from the centre of the wheel, and α is a certain angle. The last term corrects for any lack of balance of the wheel on its knife-edge. Later the sum of the first three terms on the right is called f. The balance corrections of the wheels can be found from their times of vibration.

The wheels were turned from castings of aluminium alloyed with 4 per cent. of copper, every precaution being taken in the machining to make the edge and hole concentric cylinders. The diameters are about 5 inches external and 1·25 inches internal, and the wheels weigh 250 gms.

The side of each wheel is graduated in degrees, as are the ends of the hardened steel planes on which they bear. The agate knife-edges are mounted in steel prisms. The prisms are circular quadrants in section, and are slotted to take the agates. Two grub screws in each prism enable the agate knife-edges to be very slightly advanced or withdrawn in the steel slot. The agates are finally set with sealing wax.

The planes on which the wheels rest can be accurately levelled, and then turned about a horizontal axis through a known angle. An arrestment and a pointer to read each wheel is provided. Three balancing weights, one coarse and two fine, moving on screw threads (see fig. 1, left wheel), are attached to each wheel.

The agate knife-edge, tested for straightness by Newton's rings, was made collinear with the axis of the wheel to a few ten-thousandths of an inch.

Any lateral displacement x of the knife-edge was found as it is for a balance. The following are examples of the values found for x :—

Wheel A	1.4.24	0·00030 inch.	Wheel B	29.9.24	0·00010 inch.
	29.9.24	0·00017 inch.		30.9.24	0·00026 inch.

The vertical displacement y of the knife-edge was found by measuring the diameter of the wheel and the distance of the knife-edge from the edge of the wheel by means of the Johannsen type of slip gauges and a 6-inch circular steel flat. This measurement was simple, sensitive and accurate. In wheel A, y was 0·00015 inch and in B 0·0004 inch.

The moment of inertia of the wheels was calculated from their periods of oscillation with and without a mass attached to the periphery. The above observations enabled the value of P to be calculated for each experiment.

The wire used to connect the hanging weights to the torsion wheel was 0·005 inch tungsten. The effective weight of the wire hanging vertically and in contact with a quadrant of each of the wheels was added to the sum of the weights used. These weights were gilded brass and were compared with weights standardised by the National Physical Laboratory.

Steadiness of Conditions.

It is assumed in continuous flow calorimetry that the temperature of all parts of the calorimeter are stationary. As the heat capacity of the calorimeter is 230 cal. deg.$^{-1}$, in these experiments only small changes in its temperature are permissible during a run.

When the rate of energy input to the calorimeter is equal to the rate of energy output the temperatures at different points in the calorimeter are stationary, or will gradually become so. The input is determined by the speed and magnetic field, and the output is equal to the input since the losses are effectively zero. The temperature rise for a given input depends on the rate of flow. A great deal of investigation was required before these conditions could be made sufficiently steady. Each factor contributing to steadiness will be considered in turn.

The Speed.—The speed of the motor, which changed 0·7 per cent. for a change of applied e.m.f. from 100 to 110 volts, was controlled by a centrifugal governor and hand regulated. Three equally spaced vertical lines were painted on the rotor and illuminated by a Neon tube excited by a Tinsley Drysdale electrically driven tuning-fork of frequency

50 sec.$^{-1}$. The lines on the rotor appeared to be stationary when it made 1,000 r.p.m. A hand-regulated rheostat in the field circuit of the motor enabled the speed to be held fairly constant, but it was subject (1) to frequent small variations in speed, and (2) to occasional large sudden drops in speed. The latter were found to have their origin at the commutator of the motor, and were eliminated by using brushes of graphite and copper containing a large proportion of copper, and by cleaning the commutator at short intervals. Copper and carbon dust on the commutator is the cause of momentary short circuits. The small variations in speed were removed by adding a centrifugal governor which was friction driven by a fibre wheel (as shown in fig. 7) bearing directly

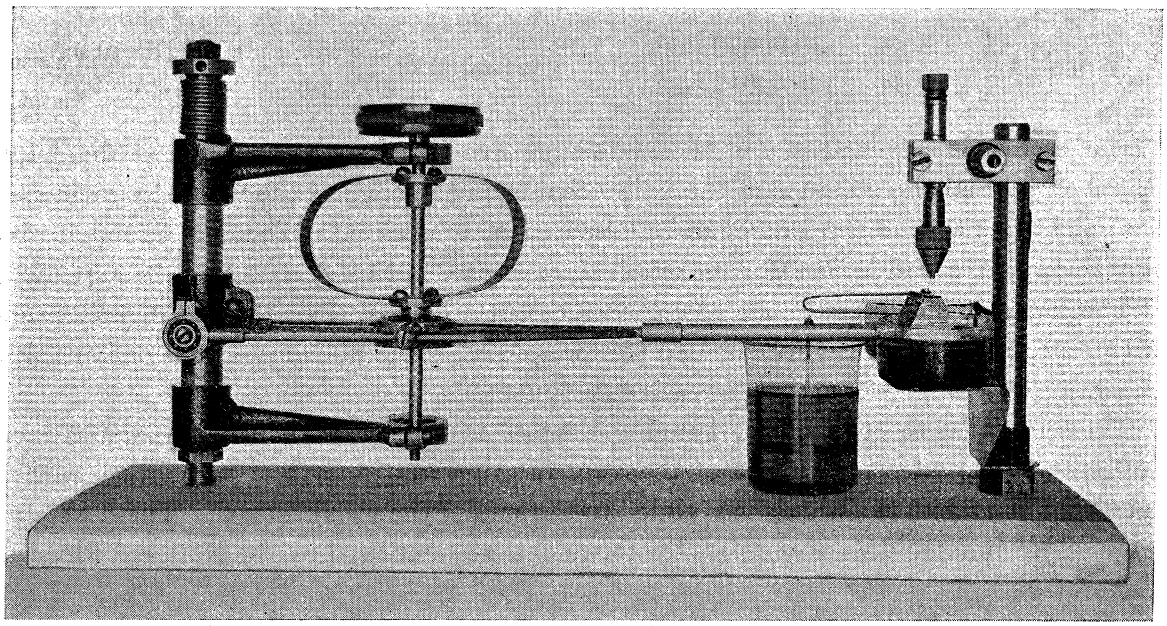

FIG. 7.—*Centrifugal Governor* (dismounted). The oil damping of the hairpin spring carrying electrical contact is shown at right.

against the rotor. When the speed rose too high, the governor closed a contact and decreased the resistance of the field winding of the motor, thereby reducing its speed. When first tried, the governor failed to function, owing to the vibration of the arm which carried the lower contact. Mr. OSBORNE eliminated these vibrations by supporting this contact by a light hairpin-shaped spring, the vibration of which was damped by vanes immersed in castor oil (see fig. 7). These measures were most successful, as is shown by the values for n given in the table on p. 90. Such small differences as there are in the turns in successive experiments may be partly due to errors in measuring the intervals of time, and in the tuning-fork which controlled the Neon tube. The constancy of the speed shown is adequate for our purpose.

Magnetic Field.—As we have seen, the stator couple depends on the rotor speed and the magnetic field, the latter being proportional to the magnetising current. This current was at first variable both because the e.m.f. of the storage battery, an old one, was variable, as was the resistance at the rotor slip rings. When the battery was replaced by a new Tudor battery of large capacity, a constant e.m.f. was obtained. Systematic experiments were needed to eliminate the variations at the slip rings. They were made part of one arm of a Wheatstone bridge, capable of carrying a current of 0·5 ampere in that arm, and the variations of their resistance measured. Brushes ordinarily used with electrical machines of graphite and copper were found to have a high and variable contact resistance. Variations 0·25 ohm occurred even with two brushes on each ring. When two fine copper wire brushes (shaped like a paint brush) in parallel were pressed on an accurately turned brass slip ring, the variations of resistance were reduced to from 1/1000 to 1/100 of an ohm in a circuit of 250 ohms resistance, and the variations in the couple to just less than 1 in 10,000.

Flow of Water.—The most serious temperature variations arise from variations in the rate of flow of the water. At first the flow was regulated by a carefully designed tap placed where the water enters the apparatus. The flow slowly decreased, and was subject to constant small irregularities, the cause of which was not at first evident to us, but later we found these fluctuations in the flow to arise from liberation in the calorimeter of dissolved air. The remedy, moving the tap to the exit side of the calorimeter, was simple enough.* The change increased the pressure of the water from about 60 to 90 cm. of mercury, and at this higher pressure the water is unsaturated with air until after it has left the apparatus. A glass capillary tube was substituted for the tap, a number of tubes with known rates of flow being made up. These changes very much improved the steadiness of the flow, but an insidious difficulty is the presence of dust or " fluff " in the water.

Distilled water was used, and after a run the water was filtered, when it was returned to the tank. The capillary resistance tube (see fig. 5) was protected against obstruction by a fine metal gauze filter through which the water passed as it entered the capillary. The steadiness of the flow is shown in the table on p. 90.

Thermometry.

The rise of temperature of the water was determined by platinum thermometers connected differentially.

Theory.—The theory of such thermometers is as follows:—The resistances R_{θ_2} and r_{θ_1} of the two thermometers in terms of temperatures on the gas scale are given by

$$R_{\theta_2} = R_0 \ (1 + A\theta_2 + B\theta_2^2),$$

and

$$r_{\theta_1} = r_0 \ (1 + a\theta_1 + b\theta_1^2),$$

then

$$R_{\theta_2} - r_{\theta_1} = R_0 - r_0 + R_0 A\theta_2 - r_0 a\theta_1 + R_0 B\theta_2^2 - r_0 b\theta_1^2.$$

* The prevention of bubbles being formed is important, not only to obtain steadiness, but to prevent the presence of air from having any appreciable effect on the specific heat of the water.

Putting

$$R_0 - r_0 = \Delta_0 \text{ and } R_{\theta_2} - r_{\theta_1} = \Delta,$$

$$\theta_2 - \theta_1 = \frac{\Delta - \Delta_0}{R_0 A} + \left(\frac{r_0 a}{R_0 A} - 1\right)\theta_1 + \frac{r_0 b}{R_0 A}\theta_1^2 - \frac{B}{A}\theta_2^2,$$

and from this we obtain, in the usual notation of CALLENDAR,

$$\theta_2 - \theta_1 = \frac{100}{(R_{100} - R_0)(1 + 0{\cdot}01\,\delta)}(\Delta - \Delta_0) + \left(\frac{r_{100} - r_0}{R_{100} - R_0} - 1\right)\theta_1$$
$$- 10^{-4}\frac{\delta}{(1 + 0{\cdot}01\,\delta)}\frac{r_{100} - r_0}{R_{100} - R_0}\,.\,\theta_1^2 - 10^{-4}\frac{\delta}{1 + 0{\cdot}01\,\delta}\theta_2^2.$$

Substituting the values of the constants,

$$\theta_2 - \theta_1 = 11{\cdot}3592\,(\Delta - \Delta_0) - \frac{\theta_1}{30} - 0{\cdot}00028\,\theta_1$$
$$+ 0{\cdot}0005853\,\frac{\theta_2 + \theta_1}{2}\,.\,\frac{\theta_2 - \theta_1}{2} + 0{\cdot}0000049\,\theta_1^2.$$

For inlet temperatures, θ_1, less than 20° the third and fifth terms on the right may be neglected for an accuracy of 0·001° C. To obtain that accuracy in $\theta_2 - \theta_1$, $(\Delta - \Delta_0)$ must be known correct to 0·0001 ohm, $R_{100} - R_0$ to about 1 in 10,000 if $\theta_2 - \theta_1$ is from 5° to 10°, and δ to 1 in 150. As δ is calculated from observations at the boiling point of sulphur, this accuracy in δ requires an accuracy of 0·15° C. in the sulphur boiling point. This degree of accuracy appears to have been attained (see p. 82). It is more difficult to estimate, however, the accuracy of the temperature observations during a run.

Mueller Bridge.—The choice of the bridge to be used was carefully considered. That the temperature rise $(\theta_2 - \theta_1)$ should be given by a single bridge reading we regarded as the most important consideration, from the point of view of both accuracy and convenience. This consideration ruled out platinum thermometers with current and potential terminals, either as used by F. E. SMITH in conjunction with the Wheatstone bridge and a mercury commutator, or as used by JAEGER and STEINWEHR with a potentiometer. Both of these methods would require four readings to calculate $(\theta_2 - \theta_1)$.

We used instead the platinum thermometers with compensating leads as originally introduced by CALLENDAR, but connected differentially as shown in fig. 8.

This enabled the Wheatstone bridge, the most sensitive of resistance measuring devices, to be used. The CALLENDAR-GRIFFITHS form of this bridge was not used as its slide wire has not the same degree of permanence and definiteness which resistance coils possess. So we decided to use the Mueller bridge as being definite in calibration, stable in its resistance, and as possessing a minimum of error from variation in resistance at contacts, *i.e.*, plugs or switches. Fig. 8 is a diagram of the Mueller* bridge used.

The effect of variation in the resistance of the sliding contacts at a, b, c, d, e, and f is greatest for b and c. The ratio of ab to ac is changed at most by 1·6 in a million when the contact resistances at b and c change by less than 0·0002 ohm.

* MUELLER, 'Bull. Bur. Stand.,' vol. 13, p. 547 (1917).

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The bridge coils are of manganin wire, and, to diminish the effect of humidity on the coils made of fine wire, the makers, Leeds and Northrup, sealed the 500-ohm ratio coils and the 10-ohm coils in oil. The bridge which we have used has proved excellent both in design and construction. The manganin wire was found to have a temperature

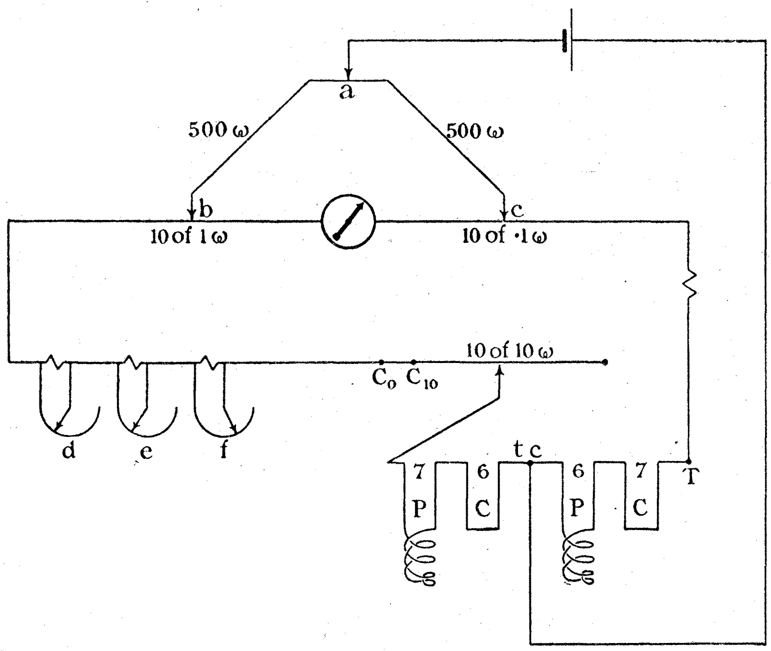

Fig. 8.—*Mueller Bridge and Connections of Platinum Thermometers.*

coefficient of about -5×10^{-6} per degree C. at 25° C. and a zero coefficient at about 20° C. We immersed all of the coils in oil contained in a copper tank round which was wound a nichrome heating coil to maintain its temperature at 25° C. The oil was stirred by a small motor driven pump.

Calibration.—Five calibrations of the 1 and 0·1 ω coils, and three of the 0·01, 0·001, and 0·0001 "coils" were made during 1924, which involved nearly 500 comparisons of resistances. In these comparisons results obtained on different days agreed to one in a million.

The bridge was also calibrated in 1920, but not with the accuracy of the 1924 experiments; a comparison of the figures obtained for the 1-ohm decade affords evidence of the constancy of the bridge, and of the manganin used in its construction.

Calibrations of 1-Ohm Coils in 1920 and 1924.

1924	0·999908	1·000070	1·000076	1·000064	1·000019
1920	0·999900	1·000060	1·000060	1·000060	1·000000
$\Delta . 10^6$	+ 8	+ 10	+ 16	+ 4	+ 19
1924	1·000038	1·000039	1·000197	1·000068	1·000171
1920	1·000080	1·000030	1·000220	1·000040	1·000200
$\Delta . 10^6$	− 42	+ 9	− 23	+ 28	− 29

The differences, Δ, of the values found in April, 1920, and June, 1924, are given in micro-ohms. The mean difference, without respect to sign, is 0·000019 ohm; taking account of sign the difference is zero, implying that the total resistance of this decade did not change in four years.

Platinum Thermometers.—The platinum thermometers were constructed by Leeds and Northrup to the general design of one of the authors. The platinum wire was wound in the usual manner on a mica cross. The gold leads fused to the platinum wire of the thermometer* were carried to the head of the thermometer, where the gold wire was silver soldered to short lengths of copper wire, which in turn were soldered to four copper posts, insulated by vulcanite and enclosed in the metal head of the thermometer. As these joints were close together, their temperature would be the same, and thermoelectric effects reduced to a minimum. The thermometers were 38 cm. long, the platinum wire coil being about 10 cm. long, and were enclosed in pyrex glass tubes, about 9 mm. external diameter. They were subject to a drying process before sealing the head with picein. The resistance at 0° C. was about 25 ohms, and the fundamental intervals at first 8·374 and 8·381 ohms: the resistance of the insulation between the thermometer and the compensating leads was about 40 megohms.

The thermometers were connected together as shown in fig. 8, in which C stands for compensating lead, and P for thermometer, and 6 and 7 for the two thermometers.

The leads to the bridge were of solid copper wire, enamel covered, further insulated by enclosing the wire in small rubber tubing. All connections, except those to the bridge, were soldered; those to the bridge were copper plugs amalgamated to copper cups. No difficulties were experienced from faulty insulation or thermoelectric effects. As mentioned later, one of the gold copper joints very slowly increased in resistance till it failed; when it was remade, no further variations occurred.

On p. 78 an expression has been given for $(\theta_2 - \theta_1)$. The numerical coefficients in this expression can be calculated when the fundamental intervals and δ are known for the two thermometers. The following observations gave these constants of the thermometers. Both were placed in ice giving $R_0 - r_0$, both in steam giving $R_{100} - r_{100}$; one in ice and the other in steam giving, with the previous observations, $R_{100} - r_0$, $R_{100} - R_0$, and $r_{100} - r_0$. Observations at the boiling point of sulphur gave δ for both thermometers.

As the methods used to make these observations partly determine the accuracy of all the thermometry, they are briefly described.

Ice Point.—Mr. E. L. Sayce, M.Sc., during the progress of our experiments, investigated the effect of impurity in ice upon its melting point, and he found the depression of the melting point was considerably larger than could be accounted for from a chemical analysis of the ice. In view of this, and as distilled water ice was not available, care was taken to use only ice which was transparent and free from any visible impurity. This ice, after washing with distilled water to remove any eutectic present, was finely

* The compensating leads were soldered to a short length of the same platinum wire.

powdered, and a Dewar flask filled with it, distilled water being added to fill up the spaces between the particles of ice. When the ice and water were stirred, a thermometer placed in the ice maintained a temperature constant and reproducible to 1/1000 of a degree.

Steam Point.—A number of hypsometers have been described, some of them of quite complicated construction. A very simple design, however, will give the temperature of condensing steam constant and reproducible to 1/1000°. This hypsometer consists of a circular brass tube (5 cm. by 50 cm.) A, closed at the bottom, with a cap at the top, and an inner concentric metal tube B, supported by light spiders from A. The water is boiled by an adjustable bunsen burner, so that a small but constant jet of steam is emitted. The steam rising inside B surrounds the thermometer and passes over the top of B and is condensed by A. B acts as a radiation shield. Little water is lost at the jet. As evidence of the effectiveness of this hypsometer, it may be mentioned that a change of barometric pressure as small as 0·001 inch of mercury, equivalent to a change in the boiling point of about 0·001° C., is detected on the thermometer a little before it shows on the barometer owing to the greater lag of the latter.

Barometer.—A Fortin barometer has been used as a working barometer in our experiments, but it was standardised by comparison (1) with a standard "contact" barometer* made for the purpose and (2) with a standard Feuss type of barometer at the Melbourne Bureau. It reads 0·0065 inch high. We are indebted to Mr. HUNT, Commonwealth Meteorologist, for allowing us access to the latter. These standard barometers agreed to 0·001 inch, which is equivalent to 0·001° C. on the boiling point of water and to 1/100,000 of the fundamental interval.

The following values for the fundamental intervals of the two thermometers have been found at the dates given.

FUNDAMENTAL Intervals of Thermometers.

Date	11.10.21	12.10.21	28.11.21	10.8.22	11.8.22
No. 7	8·3832	8·3829	8·3827	8·3824	8·3825
No. 6	8·3757	8·3755	8·3748	8·3750	8·3749
Difference	0·0075	0·0074	0·0079	0·0074	0·0076

Date	27.8.23	17.4.24	18.4.24	8.7.24	9.7.24	3.10.24	Mean.
No. 7	8·3812	8·3829	8·3820	8·3818	8·3816	8·3823	8·3823
No. 6	8·3734	8·3755	8·3739	8·3742	8·3741	8·3747	8·3747
Difference	0·0078	0·0074	0·0081	0·0076	0·0075	0·0076	0·0076

Date†	11.2.25	11.2.25	12.2.25	6.8.26	22.12.26
No. 7	8·3831	8·3827	8·3828	8·3828	8·3833
No. 6	8·6730	8·6725	8·6725	8·6746	8·6746
Difference	0·2899	0·2898	0·2897	0·2918	0·2913

* LABY: 'Journal Scien. Inst.,' vol. 1, p. 342 (1924).

† An accident to the apparatus broke thermometer No. 6 in 1925; the re-winding of it changed its resistance.

The F.I.'s show no progressive change during three years, 11.10.21 to 3.10.24, implying that the resistance of the thermometers and bridge did not alter during that period, for in the case of both thermometers the last F.I. is the same as the mean F.I. The small variations in the F.I.'s of 0·0005 ohm in the average (equivalent to 0·006° C. in 100°) no doubt arise from errors in the ice and steam-point temperatures, and in the barometer observations. Such errors would affect the F.I. of both thermometers in the same direction, which is seen to be the case, since their difference is nearly constant at 0·0076. This is confirmed by the following values of $R_0 - r_0$ observed between 29.2.26 and 26.10.26—0·75593, 0·75595, 0·75582, 0·75578, 0·75585.

It may be concluded that the F.I.'s are correct to 6 in 100,000.

The values of the F.I.'s used in the reduction of the thermometric readings (see p. 78) were, for No. 7, 8·3830, and for No. 6, 8·6746.

Sulphur Boiling Point.—Several determinations of the boiling point of sulphur were made to determine δ for the platinum used in thermometers. In our experience the sulphur boiling point is difficult to realise. We have used three hypsometers, two of them of aluminium electrically heated, and the third of pyrex glass gas heated. All of them consisted of coaxial tubes (the inner of aluminium), as described on p. 81. In one the outer aluminium tube, the lower end closed and containing the sulphur, was 4 cm. diameter and 46 cm. long. It was wound all along its length with a nichrome heating coil (insulated and covered with asbestos), most of the turns being concentrated at the lower end, experiments having been made to find the spacing which would avoid superheating. With one of these hypsometers for a power input between 250 and 310 watts, the sulphur boiled and the extreme variations of the thermometer in the sulphur vapour were 0·076° C. We are indebted to Mr. SAYCE for the skill with which he took these observations. The pyrex glass hypsometer gave a steadier temperature, and in seven observations the average deviation from the mean was 0·01° C. The following values of δ were found :—

δ of Platinum Thermometers.

Date.	Hypsometer.	δ	Date.	Hypsometer.	δ
4.8.21...	Electrically heated	1·482	26.2.24 ...	Electrically heated	1·488
16.2.21...	„ „	1·482	3.10.24 ...	Pyrex gas heated	1·504
14.2.24...	„ „	1·492	6.10.24 ...	„ „	1·500

The above are sufficiently consistent, as we have seen δ requires to be known to 1 in 150. The value of δ used was 1·485 (see p. 78). The boiling point of sulphur t at a pressure p mm. is given by

$$t = 444 \cdot 55 + 0 \cdot 0908 (p - 760) - 0 \cdot 000047 (p - 760)^2$$

on the thermodynamic scale of temperature.

Measurement of Input of Energy.

The mechanical equivalent is given by J (see p 70).

$$\pi nmg\, d_\theta f = \mathrm{J}\,[\mathrm{W}\,\{\theta_2 - \theta_1 - v\} + \mathrm{L}].$$

It will be convenient to describe in turn how the various quantities in this relation are determined.

Number of Revolutions, n.—A wheel (see fig. 5) which engages in a screw thread cut in the rotor shaft, makes one turn for 100 of the rotor, and each turn of the wheel is recorded on a drum chronograph, upon which a record is also made when the tap is switched over (see fig. 5). The turning of the tap is the beginning or the end of a run. The chronograph paper moves 8 mm. per second, or about 0·5 mm. per turn of the rotor, and it is easily read to 1 turn.

Couple, $mgd_\theta f$.—The steel torsion wire, the rubber tubing carrying the water in and out of the calorimeter, and the thermometer leads all exercise a torsional control on the stator. The position of the stator spot (read by lamp mirror and scale) when there is no couple acting on the stator will be called the stator zero. It is found in two ways, viz., (1) readings of the stator spot for, say, six different positions of the rotor* are taken, and the mean is called the static zero—static because the rotor is stationary when the reading is taken; (2) the stator spot is also read with the rotor turning slowly and demagnetised by a small current, this is called the dynamic zero. The two zeros agreed to 1 or 2 mm. on the scale, which is 1/14,000 of the deflection produced by the couple used in a heavy run (10,000 gmwt. cm.), for a couple of 10 gmwt. cm. gave a deflection ot the stator spot of 28 mm.

When the speed of the rotor was increased to full speed the stator spot was deflected some millimetres by the viscosity of the air and by the residual magnetism of the rotor. The minimum deflection (of 3 or 4 mm.) obtained with a demagnetising current gave the air drag. We have not corrected the couples for this, nor for the couple required to bend the tungsten wire. The two corrections, which are opposite in sign, about neutralise for 1000 gm. weights.

At first stator zeros taken before and after a run might differ by 20 mm., due to the yielding of the torsion wire, thermometer leads, and rubber tubes, but finally means for attaching both ends of the torsion and thermometer leads were devised which eliminated this error.

The applied couple is, $mgdf$ dyne.cm., where $d = 20{\cdot}0013 + 0{\cdot}0127$ at 20° C. The diameter of the torsion wheel was determined 20·0013 cm. by comparison with the slip gauges, which had been standardised by the National Physical Laboratory, and the wire diameter was 5 mil.; m.gm. is the mass of the weights used, and f a factor, very nearly unity, depending on the geometry of the knife-edge wheels, which is discussed on p. 74. The weights used were of brass, their mass being found by comparison with weights certified by the N.P.L.

* The residual magnetism of the rotor makes this necessary.

Measurement of Heat.

The heat output is W $(\theta_2 - \theta_1)$, and was measured as follows :—

Mass of Water, W.—The water which passes through the calorimeter during a run is collected in large glass flasks (about 5 litres capacity), the delivery tube extending down the neck of the flask, the mouth of which is loosely covered to exclude air currents. The flasks were weighed to 1/20 of a gram on a balance constructed for the purpose, the correction for buoyancy being made. The larger weights were calibrated against a standard kilogramme, and the smaller were from an accurate set by BECKER.

Evaporation.—A drying tube was not used, as it caused variations in the rate of flow. Two experiments were made to find if, in the absence of a drying tube, there were any loss of water by evaporation, the water being delivered from one flask to another, both being weighed before and after the experiment. 1,000 gms., at room temperature delivered in 3,300 seconds, showed no evaporation ; 1,080 gms. delivered at a mean temperature of about 30° in 4,000 seconds lost by evaporation 0·2 gm. In this experiment, the temperature of the water is higher, and the time during which it flows is longer than in a run, which lasts 900 seconds. We have accordingly neglected the effect of evaporation.

Temperature Rise, $\theta_2 - \theta_1$.—On p. 70 an expression has been given for $\theta_2 - \theta_1$ in terms of the bridge reading Δ, θ_1, and certain coefficients. Two dry cells were connected to the bridge, the current through the thermometers is then 1/200 ampere, and the difference in the heating of the thermometers by it is inappreciable. The sensitiveness of the galvanometer was such as to give 15 mm. deflection per 0·0001 ohm on reversing the battery, which is equivalent to 12,000 mm. per ° C. If the temperature difference was not changing too quickly, the bridge was balanced by reversing the battery in order to eliminate thermal e.m.f.'s., a reading of the bridge being taken every 15 seconds, which gives 60 readings for a 15-minute run. The mean of these is used to calculate $\theta_2 - \theta_1$ for the run, as shown by an example on p. 88. The consistency of the various experiments is evidence that the value of $\theta_2 - \theta_1$ so obtained is the real mean rise of temperature.

The temperature of the inlet water θ_1 is observed by means of a thermo-junction* attached to the outside of the platinum thermometer sheath, and forming, with another junction in ice and water in a Dewar flask, a couple whose e.m.f. was read at frequent intervals during a run on the Wolff "Thermokraftfrei" potentiometer to 1 micro volt, or to 1/40° C. This thermo-couple was calibrated against the platinum thermometers.

The determination of the heat losses is discussed on pp. 85-87.

Fluid Friction.—Part of the rise of temperature $(\theta_2 - \theta_1)$ is due to the viscosity of the water, which is heated $v°$ C. as it is forced through the copper capillary tubes of the stator. The head required to force water through the apparatus with the rate of flow used in the runs was found by a separate experiment to be 2 inches of mercury.

Hence
$$v = \rho g h / \mathrm{J} = \frac{13 \cdot 6 \times 980 \times 5 \cdot 08}{4 \cdot 18 \times 10^7} = 0 \cdot 0016° \text{ C.}$$

* The constantan wire in this couple was tested for homogeneity.

Description of an Experiment.

Before beginning a run, the whole apparatus was "tuned up" to work as efficiently as possible. The rotor brushes, the slip rings, the motor commutator and brushes were cleaned and adjusted, the chronograph and electrically driven tuning-forks examined. Any defects in any part of the apparatus meant that the high degree of steadiness, which is so essential to accuracy, would be absent, or even that some part might fail to function during the two or three hours over which the experiments extended. The tank was filled with filtered distilled water. With the water running, the zero of the stator is observed, then the weights are applied to the torsion wheel, and the knife-edge wheels and planes are turned about their axes until they make the correct angles with the horizontal for the weights to be used.

The motor would then be started, and a steadying run of an hour or more begun. During this period, the field windings of the motor and of the rotor are rising in temperature, and the speed and couple have to be continually adjusted. Meanwhile, the couple measuring the temperature difference between top of stator and room was frequently observed, the room temperature being raised by radiator and gas flames until the reading was reduced to a few microvolts, when enough heaters were left on to hold the reading steady. A lamp or heater was finally switched off just before the run, to allow for the presence in the room of a third observer. It was found possible in this way to keep the reading at a very small value throughout a run (see second last column of table, p. 90). Usually, at the end of an hour, the temperatures have become steady. A run was started when the bridge reading, *i.e.*, $(\theta_2-\theta_1)$ had been fairly constant for ten minutes.

There were three observers: One held the rotor speed constant as shown by means of the Neon tube, varying when necessary a resistance in the field circuit of the motor, and the same observer held the stator spot on its zero by adjusting a rheostat in the circuit of the field windings of the rotor. The second observer balanced and read the bridge at 15-second intervals. The third observer, watching a chronometer, switched over the tap which starts the collecting of the water and the chronograph record. He then read in turn the inlet water thermocouple, the thermocouple from which the flask loss was calculated, and the thermocouple mentioned in the preceding paragraph, which indicates the difference in temperature of the top of the calorimeter and the air. The same observer kept a general oversight of the apparatus, cleaning the motor commutator periodically.

To economise time and to have a check on the reproducibility of the results, four consecutive runs of 15 minutes have been taken in the later experiments. The second time the tap is switched the first run ends and the second begins. No difficulty arises about collecting the water, the collecting flasks being under the two taps alternately.

At the end of a run the stator zero is re-observed to see that no alteration has occurred in it.

Heat Losses.

How the heat losses have been made a minimum has been stated on p. 72.

On entering the stator, the water flows through the narrow annular space between the

" inlet " platinum thermometer and the inner wall of the vacuum jacket. The ingoing water is about 2° C. below air temperature and so rises slowly in temperature (at about $\frac{1}{14000}$° C. per cm.) as it flows past the thermometer coil, which is about 10 cm. long. We take the temperature recorded by a thermometer as that of the water, which is level with its middle coil.

The same applies, *mutatis mutandis*, to the outlet thermometer.

It is convenient to take a plane at the common level of the middle coils of the two thermometers as part of the " boundary " of the calorimeter ; so that in estimating heat loss we need consider only the heat transfer through this boundary and through the flask below it. Heat transfers that are purely circulatory inside this boundary do not concern us.

Three sources of heat loss (or gain) are to be allowed for :—

1. Loss through flask walls.
2. Loss through mouth of flask.
3. Loss through thermometer jackets.

Loss through Flask Walls.—As previously stated, the top of the stator was at room temperature, while the bottom, from which the water leaves, is its hottest part. The loss through the flask walls can be taken as proportional to the mean excess of stator temperature above that of the air. This temperature excess was determined at intervals throughout a " run " by means of a thermocouple, of which one junction was attached to the outside wall of the flask and the other placed on the inner wall at a level with the middle of the heat radiating surface (see fig. 3, F). In order to test the flask, and determine the heat transfer per degree excess, it was filled with water at about 2° C., corked, and left for about a day immersed in a tank of water at room temperature. Temperatures were read frequently both for the water inside and that outside the flask, and the total gain was found to be 8·2 calorie per hour per degree temperature difference. Allowing for conduction down through the air 0·81, and along the glass wall 0·72, calorie per hour (calculated values), this gave for the heat transfer through the flask walls

$$6{\cdot}7 \text{ cal. hr.}^{-1} \text{ deg.}^{-1},$$

or in terms of the thermocouple reading

$$0{\cdot}173 \text{ cal. hr.}^{-1} \text{ microvolt}^{-1}.$$

This correction is about 3 in 10,000 in a run.

Loss through Mouth of Flask.—This was reduced to a negligible amount as described on p. 85.

Loss through Vacuum Jackets surrounding the Thermometers.—The efficiency of the insulation by the vacuum jackets was tested as follows ; A copper-constantan thermocouple, of which one junction was placed approximately at the centre of the inner wall, the other at the centre of the outer, determined the temperature drop between the inside and outside of the jacket. A thin Eureka wire was placed axially in the jacket, attached to stouter leads passing through corks at the ends, and the inside was filled with cork dust to prevent longitudinal flow by convection. The whole was immersed in water at room temperature and left for some hours with a current through the axial wire in order to attain thermal equilibrium. Results on two days gave for the whole loss from the jacket :—

$$0 \cdot 00324 \text{ cal. sec.}^{-1} \text{ deg.}^{-1},$$

$$0 \cdot 00327 \text{ cal. sec.}^{-1} \text{ deg.}^{-1},$$

giving a mean of $0 \cdot 00325$ cal. sec.$^{-1}$ deg.$^{-1}$ or $11 \cdot 8$ cal. hr.$^{-1}$ deg.$^{-1}$ for a length of about 20 cm.

That the axial heat flow in this test is unimportant is shown thus :—

The heat supply to the wire is $0 \cdot 00092$ cal. sec.$^{-1}$ cm.$^{-1}$, the conduction along it is $0 \cdot 000067$ cal. sec.$^{-1}$ deg.$^{-1}$ cm., and the radial loss from it is $0 \cdot 00016$ cal. sec.$^{-1}$ deg.$^{-1}$ cm.$^{-1}$, as stated above. If θ is the temperature of the wire at a point x cm. from its centre, and θ_A is the air temperature, then

$$0 \cdot 00092 + 0 \cdot 000067 \frac{d^2\theta}{dx^2} - 0 \cdot 00016 (\theta - \theta_A) = 0.$$

Solving this and plotting the temperature distribution along the wire shows that the temperature of the wire is sensibly uniform except near its ends.

In applying this to a J experiment, the stator top and the air at the top of the flask are at the same temperature $\theta_A{}^\circ$, so that the inlet water at $\theta_1{}^\circ$ will receive heat at a rate depending on $\theta_A - \theta_1$, and the outlet water at $\theta_2{}^\circ$ will lose heat at a rate depending on $\theta_2 - \theta_A$. The net loss is proportional to $\theta_1 + \theta_2 - 2\theta_A$. Some of this loss, from the lowest part of the jacket, just above the stator, goes to the stator itself, and is therefore not to be reckoned as a loss from the calorimeter. From symmetry, one may assume this to apply below a horizontal plane midway between stator and mouth of flask, $3 \cdot 5$ cm. below mouth of flask. This plane is 5 cm. below the plane marking the "calorimeter boundary," so that the effective loss is $(\theta_1 + \theta_2 - 2\theta_A) \times 5 \times 0 \cdot 59$ cal. hr.$^{-1}$. Since the mean excess $\dfrac{\theta_1 + \theta_2}{2} - \theta_A$ was less than 1° C. in the actual runs, this involves a loss of not more than 6 cal. hr.$^{-1}$, and this may be neglected in a run developing 88,000 cal. hr.$^{-1}$.

Details of an Experiment.

The following gives the complete details including the calculation of a typical experiment :—

Experiment of 30.7.26.

Input—

Suspended mass ..	1000·000 gm.	Initial resting point ..	259·6
Equivalent of wire on quadrants	0·029	Final resting point ..	253
Mass of vertical wire	0·195	Mean	256·3
Torsion correction ..	0·135	Spot held at	259
Balance correction ..	− 0·231	Torsion correction ..	2·7/20
Buoyancy correction	− 0·143	Wheel A period 15 sec.	

Correction $\dfrac{28 \cdot 9 \sin 34^\circ}{15^2}$ .. $= 0 \cdot 071$

Corrected mass m .. 999·985 gm.

Number of turns n .. 15023

Correction for centering of knife-edge f .. 1·00013

$\pi g d_{15 \cdot 4}$ .. .. 61610·6

Corrected input $= \pi g d_{15 \cdot 4} f n m$

$= 92568 \,.\, 10^7$ erg.

Wheel B period 10 sec.

Correction $\dfrac{28 \cdot 9 \sin 34^\circ}{10^2}$ .. $= 0 \cdot 161$

Correction for balance .. $= 0 \cdot 231$ gm.

Output—

Weighings.

Flask and water, left, 5,033·7 ; right, 5,033·25		5,033·47
Flask ,, 845·45 ,, 845·25		845·35
Water		4,188·12
Buoyancy correction		4·45
Mass of water	W =	4,192·57

Bridge Readings (at 15-sec. intervals)—

	1·2578	1·2570	1·2566	1·2567	1·2570	1·2570
	3	0	7	70	68	4
	2	2	8	4	9	4
	0	2	5	1	X	1
	1	1	5	67	70	4
	1	3	5	7	2	3
	1	2	9	8	0	3
	1	69	7	9	2	5
	2	6	71	X	3	7
	1	7	69	71	1	7
Means	1·2572	1·25702	1·25672	1·25694	1·25705	1·25738 ohms.

$$\theta_2 - \theta_1 = 11 \cdot 3592\,(\Delta - \Delta_0) - \frac{1}{30}\theta_1 + 0 \cdot 0005853\,\frac{\theta_2 - \theta_1}{2}\,\frac{\theta_2 + \theta_1}{2} - 0 \cdot 00028\,\theta_1 + 4 \cdot 92 \,.\, 10^{-6}\,\theta_1^{\,2}.$$

Mean bridge reading	..	1·25705 ohms.
Correction for calibration	—	0·00021
		1·25684
Both thermometers in ice Δ_0	..	0·75535
$\Delta - \Delta_0$	..	0·50149
$11{\cdot}3592\ (\Delta - \Delta_0)$	..	5·6965
$- 13{\cdot}23/30$	$= -$	0·4410
$0{\cdot}0005853 \times 2{\cdot}64 \times 15{\cdot}87$	$= +$	0·0246
$-0{\cdot}00028 \times 13{\cdot}2 + 4{\cdot}9.10^{-6}\ 174$	$= -$	0·0028
Rise of temperature of water	..	5·2773
Correction for friction	—	0·0016
Corrected rise	..	5·2757° C.

$W\ (\theta_2 - \theta_1 - v) = 4192{\cdot}57 \times 5{\cdot}2757$

Uncorrected output = 22119 calorie

Flask loss = 5 ,,

Corrected output = 22124 ,,

$$J = 92568\,.\,10^7/22124 = 4{\cdot}1841_5\,.\,10^7 \text{ at } 15{\cdot}87^\circ \text{ C.}$$

Tabulation of Experiments.—The detailed results of the 23 runs are shown in the table below. The same symbols are used as in the formula p. 70, $\theta_2 - \theta_1 - v$ includes the correction v for viscous heating; the column headed "Brass Couple" indicates how closely the air temperature was adjusted to that of the stator top; readings are given in microvolts, 40 microvolts being equivalent to 1° C.

The individual quarter-hour experiments are in good agreement with each other, except on 11.8.26, when there were some fluctuations in speed, due to slipping of the belt drive. The mean for each set of four continuous experiments was calculated, and in deducing the final result, these were weighted, roughly according to the steadiness of conditions for the set. These weights are shown in the last column.

The weighted mean of the experiments is

$$1 \text{ calorie at } 16{\cdot}7^\circ \text{ C.} = \mathbf{4{\cdot}1841 \times 10^7} \text{ erg.}$$

with a probable error of $\pm\ 0{\cdot}0001 \times 10^7$. This differs little from the equally weighted mean $4{\cdot}1840\,.\,10^7$.

Using CALLENDAR's values for the variation in specific heat of water, this gives:

$$15^\circ \text{ C. calorie} = 4{\cdot}1860\,.\,10^7 \text{ erg.}$$
$$20^\circ \text{ C. calorie} = 4{\cdot}1809\,.\,10^7 \text{ erg.}$$

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SUMMARY of Experiments.

Date.	m.	n.	Input joule.	θ_1	$\theta_2-\theta_1-v$.	W.	Flask loss.	Output.	Brass couple	J.	J mean.	Mean temp.	Weight.
28.7.26	1000·175	15021	92574	13·28	5·3899	4103·0	6·6	22121	3	4·1849	4·1848	15·97	2
		15019	92561	13·28	5·3918	4100·5	6·6	22116	0	4·1852			
		15021	92574	13·28	5·4040	4092·8	6·6	22124	1	4·1843			
30.7.26	999·985	15023	92568	13·23	5·2757	4192·6	5·1	22124	—2	4·1842	4·18425	15·88	4
		15016	92525	13·24	5·2920	4177·5	5·1	22112	—3	4·1843			
		15025	92582	13·25	5·2866	4184·2	5·1	22127	—3	4·1842			
		15024	92574	13·25	5·2450	4217·2	5·1	22124	—2	4·1843			
3.8.26	999·900	14989	92349	13·51	5·2763	4181·5	5·9	22069	1	4·1846	4·1844	16·15	2
		14991	92359	13·51	5·2898	4170·7	6·1	22068	—3	4·1852			
		14985	92324	13·51	5·2860	4174·6	5·8	22073	—2	4·1827			
		14980	92294	13·51	5·2772	4177·8	5·9	22053	2	4·1851			
11.8.26	999·875	15003	92436	14·10	5·2326	4224·4	5·3	22108	1	4·1811	4·1833	16·74	1
		14988	92344	14·12	5·2266	4223·1	5·4	22078	2	4·1827			
		15001	92425	14·14	5·2377	4213·5	5·4	22074	0	4·1871			
		15005	92449	14·15	5·2388	4218·5	5·4	22105	0	4·1823			
13.8.26	999·858	14999	92407	12·99	5·2729	4188·2	5·6	22090	1	4·1838	4·18425	15·62	2
		15004	92437	12·99	5·2666	4193·4	5·6	22091	0	4·1850			
		15002	92425	12·99	5·2683	4192·3	5·7	22093	1	4·1841			
		15009	92468	12·99	5·2720	4191·3	5·6	22103	—1	4·1841			
12.11.26	999·838	15035	92637	17·75	5·4047	4097·1	6·3	22150	—2	4·1824	4·1832	20·50	2
		15041	92674	17·79	5·4090	4094·0	6·3	22150	—2	4·1840			
		15032	92619	17·85	5·4189	4085·3	6·2	22144	—3	4·1827			
		15037	92650	17·87	5·4189	4085·7	6·2	22146	—2	4·1837			

Weighted Mean $4\cdot1841\times10^{7}$ at $16\cdot67°$ C.

Discussion of Results.

Comparison with Previous Results.—A critical discussion of the previous work has been given by the senior author.*

REYNOLDS and MOORBY's result was in terms of the mean calorie. Reduced to the 20° C. calorie it is 4·1758 . 10^7, but this is of little value as the value of the ratio 1 mean calorie : 1 calorie at 20° C. is not known accurately for reasons previously stated (see p. 67).

ROWLAND's thermometry, as already mentioned, makes his value of the 20° calorie 4·1822 . 10^7 erg. uncertain.

As to the electrical experiments we think it is not possible to say what the absolute values were of the Clark cells used by GRIFFITHS and by SCHUSTER and GANNON. This leaves the following values :—

CALLENDAR and BARNES	4·1795 × 10^7
BOUSFIELD and BOUSFIELD	4·1767 × 10^7
JAEGER and STEINWEHR	4·1821 × 10^7

in erg per 20° C. calorie.

The absolute value of the e.m.f. of the cells used by CALLENDAR and BARNES introduces some uncertainty in re-calculating their results. The absolute values of the electrical standards used by JAEGER and STEINWEHR are accurately known. It is to be concluded that the electrical experiments give about 4·181 × 10^7 erg. as equivalent to the 20° calorie.

Conclusion.

1. A summary of this paper is given on p. 63.
2. A direct determination of the mechanical equivalent of heat gives the value

$$1 \text{ calorie at } 16{\cdot}7^\circ \text{ C.} = \mathbf{4{\cdot}1841 \,.\, 10^7} \text{ erg.,}$$

which is equivalent to

$$1 \text{ calorie at } 15^\circ = 4{\cdot}1860 \,.\, 10^7 \text{ erg.}$$
$$1 \text{ calorie at } 20^\circ = 4{\cdot}1809 \,.\, 10^7 \text{ erg.}$$

on the scale of temperature of a platinum thermometer whose constants are determined at 0°, 100°, 444·55° on the thermodynamic scale.

3. This value agrees with values found by indirect electrical experiments and so confirms the present accepted absolute values of the practical electrical units.

In conclusion, we wish to express our indebtedness to Mr. J. L. OSBORNE, who shared in taking the observations, and whose mechanical skill in the construction of the apparatus contributed greatly to its success.

* 'Proc. Phys. Soc. London,' vol. 38, Part 3, p. 169 (15.4.26).

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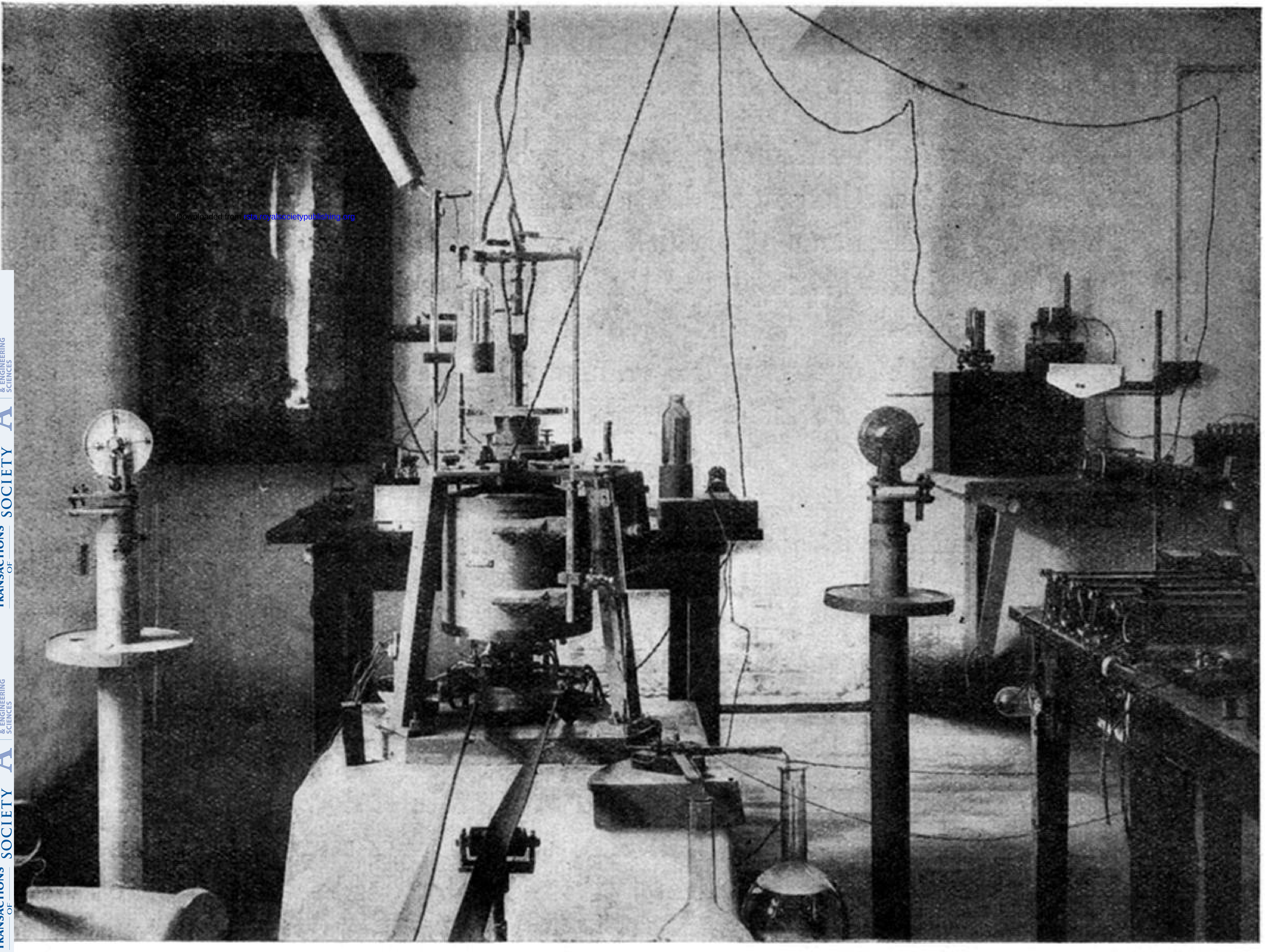

FIG. 1.—*Mechanical Equivalent of Heat Apparatus.* Induction dynamometer in centre of photograph.

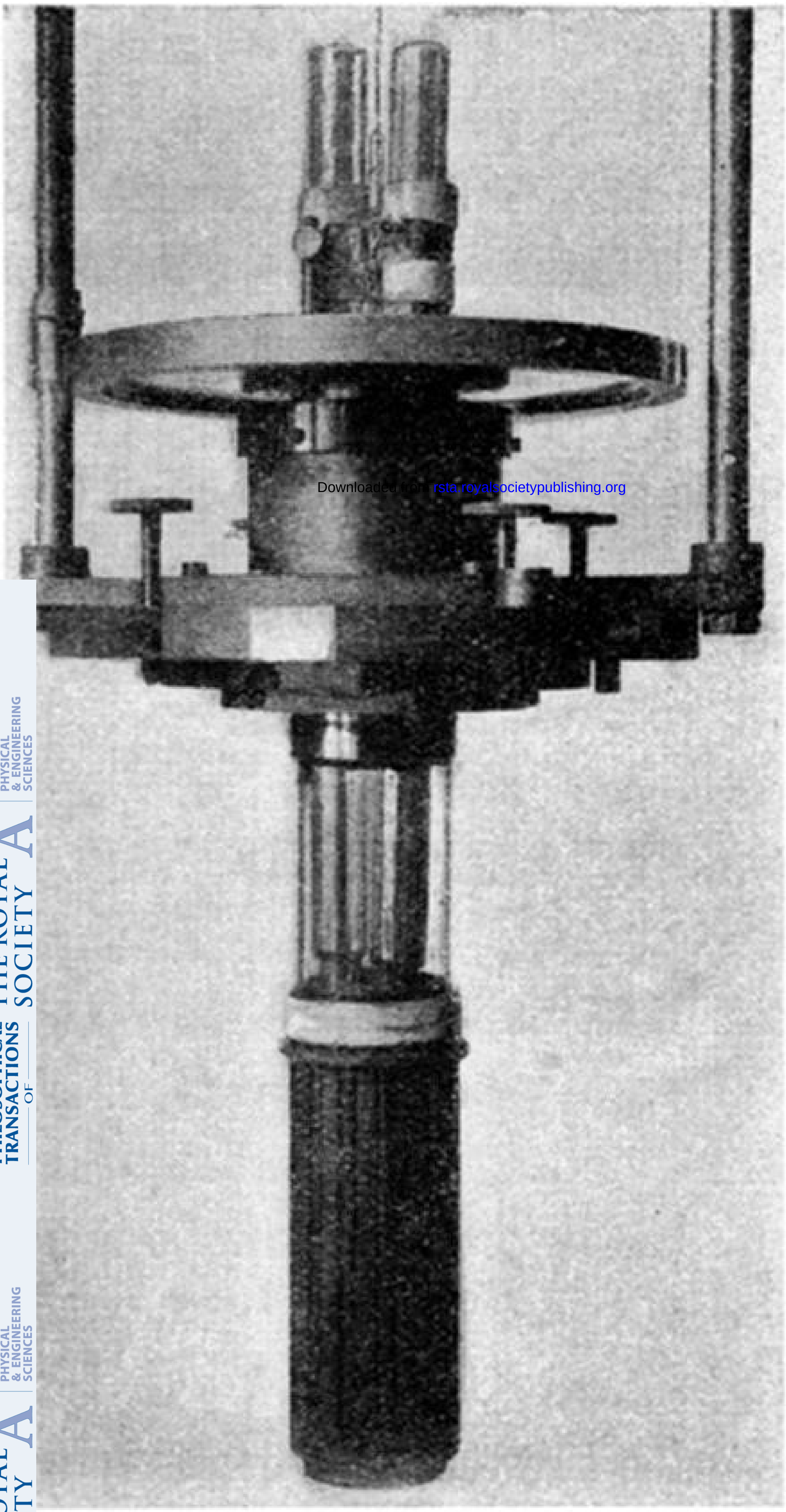

FIG. 4.—*The Stator.* From lower end upwards: squirrel-cage armature, and copper tubes, vacuum jackets, levelling screws, outer case of ball bearing, torsion wheel.

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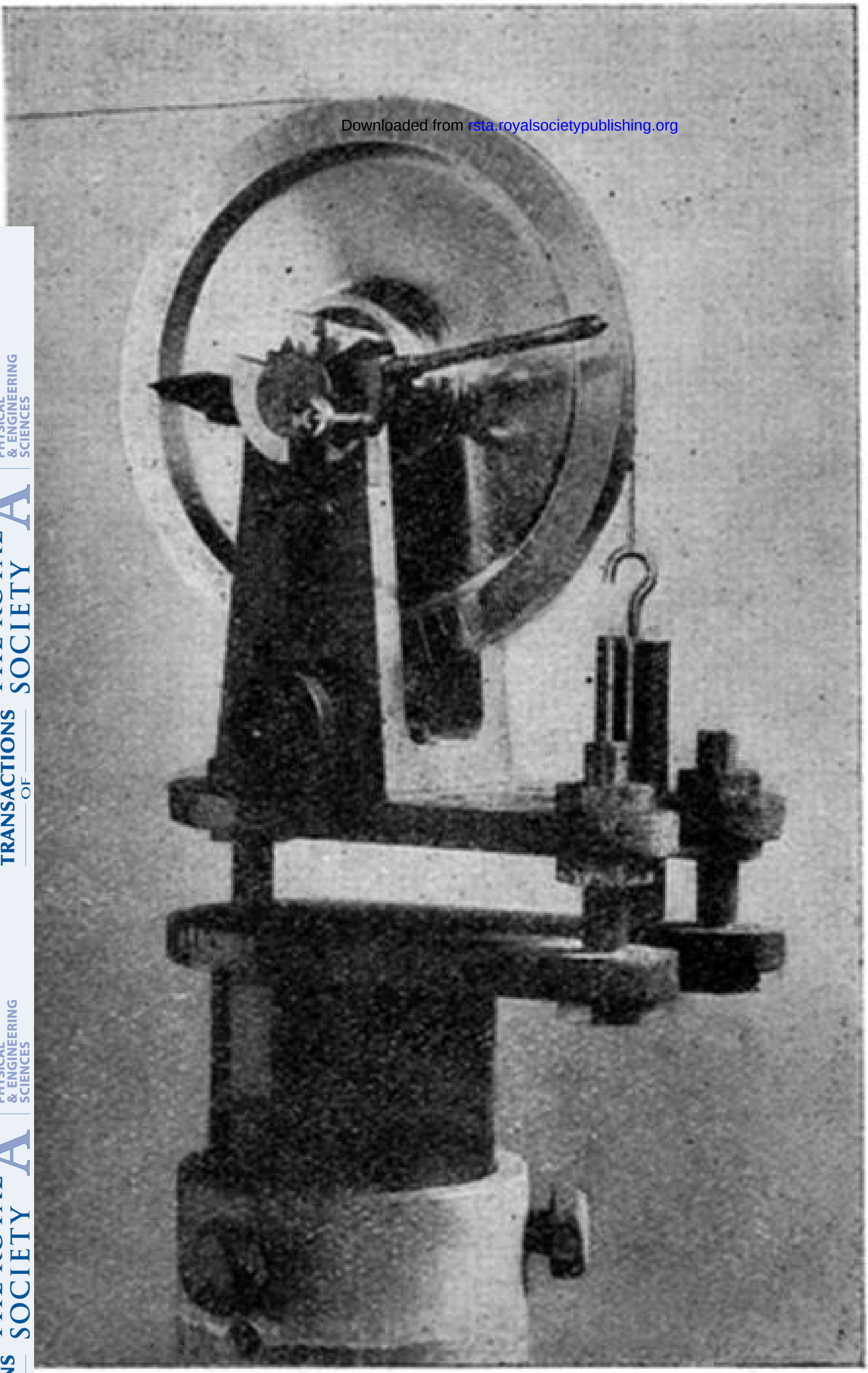

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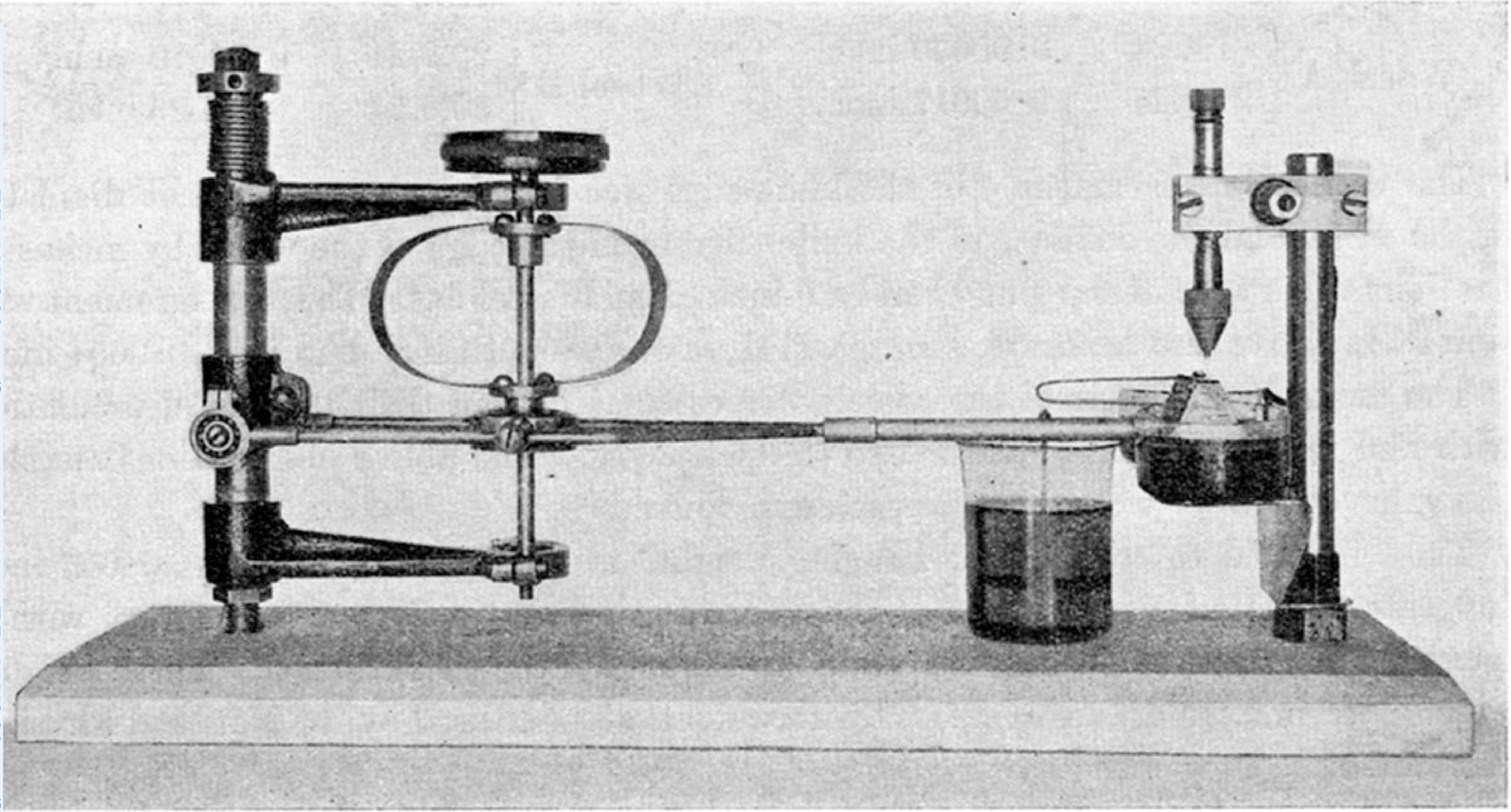
'IG. 6.—*Agate knife-edge Side Wheels.* The other face of a wheel is shown on the left of fig. 1.



IG. 7.—*Centrifugal Governor* (dismounted). The oil damping of the hairpin spring carrying electrical contact is shown at right.